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FIFTEENTH CONFERENCE OF THE INDIAN MATHEMATICAL SOCIETY

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PROGRAMME OF THE CONFERENCE SESSION

Monday, the 22nd December 1947

8–30 to 10–30 A.M.	Opening Session.
	Conference inaugurated by Sree Sree Sree Dr. Vikrama Deo Varma Garu, Maharajah of Jeypore, *Patron*.
	Welcome Address, by Dr. C. Ramalinga Reddy, Vice-Chancellor, Andhra University, and Chairman of the Reception Committee.
	Reading of Messages received, wishing success to the Conference, by Dr. A. Narasinga Rao, Local Secretary.
	Secretary's Report, by Dr. T. Vijayaraghavan, Secretary of the Society.
	Presidential Address, by Prof. M. Raziuddin Siddiqi, President of the Society.
	Announcements, by Prof. S. Bhagavantam.
10–30 A.M. ..	A visit round the University Laboratories, Meteorological Station and the Mathematics Exhibition.
2–15 to 5 P.M. ..	Visit to the Vizag Harbour and Boating Excursion.
5–15 to 6 P.M. ..	Civic Reception and "At Home" at the Vizagapatam Town Hall.
6–15 P.M. ..	Public Lecture at the Town Hall, by Sir C. V. Raman, KT.

Tuesday, the 23rd December 1947

8–30 to 10–30 A.M. ..	Symposium on Theoretical Physics inaugurated by Sir C. V. Raman, KT., with an address on "The Eigen Vibrations of Crystal Lattices".
10–30 A.M. to 12–30 P.M.	Reading of Papers.
2–30 to 5 P.M. ..	Visit to "Scindia Steam Navigation Co.'s Shipyards".
5 to 6 P.M. ..	"At Home" at the Shipyards.
6–15 to 7–15 P.M. ..	Discussion on the Teaching of Mathematics (University Buildings).
8–30 P.M. ..	Business Meeting of the Society.

Wednesday, the 24th December 1947

8-30 to 10 A.M.	..	Symposium on "Methods of Testing Hypotheses", inaugurated by Prof. U. Sivaraman Nair.
10 to 10-45 A.M.	..	Lecture by Rev. Father Racine on "Determinants in a Skew Field".
10-45 A.M. to 12-30 P.M.		Reading of Papers.
2-15 to 3-30 P.M.	..	Symposium on "Theory of Measure", inaugurated by Prof. F. W. Levi.
3-30 to 5 P.M.	..	Annual Meeting of the Benares Mathematical Society.
5 to 6 P.M.	..	Reception Committee "At Home" (Erskine Square).
6-15 P.M.	..	Public Lecture, by Sir K. S. Krishnan, KT.

Thursday, the 25th December 1947

8 A.M. to 12 Noon	..	Visit to Simhachalam Hill and Temple.

The Opening Session was held in the Geology Lecture Hall, Erskine College, and all other meetings in the Physics Gallery of the Jeypore Vikrama Deo College of Science.

గణితశాస్త్ర పండితోత్తములారా !

గణితశాస్త్రమును దాని యాధారమైన జ్యోతిష శాస్త్రమునందలి స్కంధ త్రయమును భారత దేశమునందుఁ గలిగి యున్నత స్థితిని బొంది యితర దేశములకు వెళ్ళినట్లితిహాసమువలనఁ దెలియును. షహాజహాన్ మొదలగు మొగల్ రాజుల కాలమునందును జయసింగ్ మొదలగు స్వదేశ ప్రభువుల కాలమునందును ఖగోళ శాస్త్ర మీదేశములో నభివృద్ధి జెందినట్లు నిదర్శనములుగా నున్న అబ్జర్వేటరీలు (Observatories) జైపూరునందును ఢిల్లీయందును వెలయుచున్నవి. భాస్కరాచార్యులు, వరాహమిహిరాచార్యులు, ఆర్యభట్టులు మొదలగువారి పరిశోధనలు మీ యందఱకుఁ దెలిసియున్నవియె. ఇట్టి జ్ఞానాకరమును గలిగిన మన దేశములో గణితశాస్త్రజ్ఞులనేకు లుద్భవించి లోకోత్తరములైన పరిశోధనములు గావించుటకు సువిధలు గలవు. మన దేశముయొక్క సంస్కృతి నిష్కామ కర్మ ప్రాధాన్యముతోడను "కర్మణ్యేవాధికారస్తే" యను విధితోడను నియమింపఁబడి యుండుటచే ధనాపేక్ష లేక కేవల విజ్ఞానాభివృద్ధిని జేయఁ దలంచుకొన్నవారికి నట్టివారు రచింపఁ బోవు గ్రంథములకు నీ దేశము యుక్తమనుట నిక్కువము. కనుక మీ విజ్ఞానమును గణితశాస్త్ర సంబంధమగు మీ Indian Mathematical Society యును అభివృద్ధి చెందఁగలవు. అందులకుదాహరణముగాఁ గొలఁది సంవత్సరముల క్రిందటనే మా ప్రదేశమునందు జన్మించి చిన్నతనముననే జగద్విఖ్యాతి గాంచి స్వర్గతుఁడైన శ్రీనివాస రామానుజమును గైకొనవచ్చును.

మీ సంఘము 1907 వ సంవత్సరములో స్థాపింపఁబడినట్లు వింటిని. ఈయది దానియొక్క పంచదశ ద్వివార్షికోత్సవము. భారతదేశస్థ గణితశాస్త్రజ్ఞు లిక్కడకు విచ్చేసి యున్నారు. ఈ యాంధ్ర విశ్వకళా పరిషత్తు బాల్యావస్థయందున్నది. అట్లయినను విజ్ఞానాభివృద్ధి కొఱకుఁ బాటుపడి దేశాభివృద్ధికిఁ దోడ్పడవలెనని సంకల్పించుకొన్నది. ఈ పంచదశ ద్వివార్షికోత్సవము స్వాతంత్ర లబ్ధ భారత దేశమునందుఁ బ్రథమ సమావేశము.

మా యాహ్వానము నంగీకరించి యీ సమ్మేళనమును మా కళాశాలలో జరుపుటకు Indian Mathematical Society సాంఘికు లంగీకరించుటవలన మాకు గణితశాస్త్రములోఁ గొంత ప్రాముఖ్యతయు భావిగణితశాస్త్రాభివృద్ధికి మార్గమును గల్పించియున్నారు. అందులకై మా ధన్యవాదములు. ఈ విషయములో నధ్యాపకులును విద్యార్థులును వారికి నిర్ణయింపబడిన కార్యములను లెస్సగా నిర్వర్తింతురని నమ్ముచున్నాను.

ఇక్కడకు విచ్చేసిన ఘనులీ రెండు మూడు నాళ్ళలో జరుగు సభలలోఁ బాల్గొని గణితశాస్త్ర సంబంధ విషయములను జర్చించి లోక కల్యాణమునకుఁ

దోహదపడవలయునని కోరుచు నీ పంచదశ ద్వైవార్షిక సమ్మేళన కర్తవ్య కార్యక్రమ
మును ప్రారంభించుచున్నాను.

శుభం భవతు

విశాఖపట్టణము,
ది॥ ౨౨-౧౨-౪౭.

శ్రీ విక్రమ దేవవర్మ

Translation of the above Speech

LEARNED SAVANTS IN THE SCIENCE OF MATHEMATICS,

History teaches us that the science of Mathematics, and those of Astrology and Astronomy based thereon, had their beginnings and attained a high state of development in India, and then travelled to other countries. The observatories which are to be seen even at the present day at Delhi and Jaipur bear witness to the progress of astronomical studies under the patronage and initiative of the Moghul Emperors like Shah Jehan and Indian potentates like Jai Singh. The investigations of men like Bhaskaracharya, Varahamihira and Aryabhatta are well known to all of you. With such rich traditions, it is to be expected that mathematicians should arise in large numbers in our country and make outstanding contributions to the development of the subject. Our culture is imbued with the ideal of work without a thought of the fruits thereof, and lays down the law "Your right extends only to the work"; hence our land is a fit arena for great investigators working for the love of knowledge without a thought of self or of material possessions. Hence also it is a land in which the Science of Mathematics and the Indian Mathematical Society should thrive. In support of the above view, one may cite the case of Srinivasa Ramanujan, born in this country, who though he died young attained worldwide recognition.

I understand that your Society was established in 1907. You are now celebrating its Fifteenth Biennial Conference. Mathematicians from all over India are assembled here. The Andhra University is still in its infancy. But it is firmly resolved to work for the progress of scientific studies and thus to serve the needs of the country. This Fifteenth Biennial Conference is the first to be held by you in India after its attainment of Freedom.

By graciously accepting our invitation and holding this Conference here, the Indian Mathematical Society has given us a prominence in the mathematical world and stimulated mathematical studies in this place. We are most grateful to you for this. I am confident that the teachers and students assembled here will, by carrying out the duties assigned to them, contribute to the success of the Conference.

With the wish that during the ensuing 2 or 3 days, the savants assembled here will, by taking part in the deliberations and discussions, contribute to the welfare of the world, I have great pleasure in declaring this Conference open.

May the Auspicious happen.

Vizagapatam,
22-12-47.

SRI VIKRAMA DEVA VARMA.

Welcome Address

BY

DR. C. RAMALINGA REDDY, M.A. (CANTAB.), HON. D.LITT. (ANDHRA),
Vice-Chancellor, Andhra University and Chairman of the Reception Committee.

DELEGATES TO THE MATHEMATICAL CONFERENCE, LADIES AND GENTLEMEN,

On behalf of the Andhra University, its Officers, authorities and Faculties, I beg to extend to you a very hearty welcome. Our Chancellor has sent his greetings and good wishes the full text of which will be read out to you in due course. Our distinguished Pro-Chancellor, scholar, poet and nobleman of high degree, the Maharajah of Jeypore, has done the honours of the University by inaugurating this Session. As the next figure in the hierarchy, I join my superiors and all my colleagues in wishing you a pleasant and successful time in the Campus of the Andhra University.

Vizagapatam is a town of destiny in Andhra Desa. We expect ship-building on a large scale and naval training and many other organisations of peace and war to be in abundant operation in the near future, for all of which Science and Applied Sciences are a necessary element; and these Sciences in their turn depend on mathematics for their progress. And so it is but fitting that this Session of the distinguished mathematicians of India should be here at this time when Vizagapatam is at the commencement of a new era of special significance for the defence and prosperity of our country.

Let me straightaway confess that my knowledge of mathematics is an absolute zero. For that very reason I have great respect for that subject and for those who are engaged in its cultivation and development. I need not tell you that ignorance is one of the firmest roots of reverence. This has led me to glance at the part played by India in the History of Mathematics. Our ancients are credited with the Decimal System of Notation and other inventions and discoveries which have made modern mathematics possible. Varaha Mihira in his *Suryasiddhanta* explained eclipses and gave astronomy the dynamic aspect of an exact science. In the days when I was compelled much against my will and inclination to study mathematics in the old F.A. classes, some problems in Algebra used to be set from Bhaskaracharya's *Leelavathi*. We have a big heritage which has received development in our own times by the genius of Srinivasa Ramanujan and others.

Mathematics appears to be a specialty of the Indian mind with its inherent propensities for abstract thinking of a high order. There are several luminaries in the present gathering and let me hope that they will spare no pains in putting free India on the world map of mathematics in the same, if not a more glorious, position which it held in the ages that have gone by.

Remember, in those ancient times we had no competitors in the world; today we have to hold our own against a world alert, ahead and dynamic.

The Andhra University has not yet had a long history and in a sense it is one of the younger Universities of our country. We are, nevertheless, trying to establish healthy traditions and a research atmosphere, so very necessary for a University. We have here a department of Pure Mathematics, which is nearly as old as the University itself (*i.e.*, as young), and we are, in a sense, pioneers in our land in respect of a Department of Mathematical Physics. These two departments work in close co-operation with other branches of science, such as Physics and Chemistry, Geology and Technology; and I hope during your stay here, you will see how we are faring in these lines and aid our development with your advice. I trust that at the end of your labours your hosts will be able to feel indebted to you as deeply for your inspiration and help as for the pleasure of your company. I hope you will appreciate our efforts even if our achievements are inadequate and excuse the discomforts to which you have been put to by lack of proper accommodation. It is not often that so many savants gather in one place; and this is indeed the first occasion when so many eminent mathematicians from all over India have come to our University Campus. To some of you this University is not altogether new. To others I would like to say that we all feel highly honoured by your visit. Our laboratories, libraries, museums and so on, are open and you are welcome to go round and enliven our young men who work there by your company and conversation. We are young and need more guidance from you as we need material help from the Government.

I hear that the Indian Mathematical Society was founded in 1907 and has been steadily progressing and rendering useful service in the cause of mathematics in India. Its journals and other publications should be serving a real need in our country and helping the leading research workers; but I think the Society should play even a more prominent and decisive part in the future of India and give a lead to the organisation of mathematical studies. From your programme, I notice that you are devoting some time for discussing the teaching of mathematics besides different aspects of the subject itself. Mathematics is, in the main, an abstract subject and many people complain that its teaching is even more abstract in the hands of bad teachers. A great deal of reform appears to be necessary in this direction and attempts should be made to raise the standards of teaching and the standard of teachers of all grades. Mathematicians have always been a peculiar lot to deal with. A very great authority in your own subject appears to have remarked: "It is easier to square the circle than to get round a mathematician." Nevertheless, I hope that you will devote your time to this important aspect of raising standards by devising schemes of a practical nature.

Let me in conclusion express my best wishes to you all and extend a most cordial welcome.

Messages Received Wishing a Successful Session of the Conference

From HIS EXCELLENCY REAR-ADMIRAL THE EARL OF MOUNTBATTEN OF BURMA, K.G.P.C., G.M.S.I., G.M.I.E., G.C.V.O., K.C.B., D.S.O., D.C.L, LL.D., A.M.I.E.E., Governor-General of India, New Delhi.

I have great pleasure in sending this message on the occasion of the 15th biennial conference of the Indian Mathematical Society which is being held this year at Waltair under the auspices of the Andhra University.

Mathematics owes much to the contributions made by India both in the past and in modern times. It is in the fitness of things, therefore, that yours is the oldest scientific society in the country.

Mathematics being the basic science, Indian Mathematicians have a special responsibility since no fundamental progress in the sciences is possible without the help of mathematics. Even in the domain of the biological and medical sciences, mathematics is finding increasing applications, and perhaps it will not be too much to say that future progress in these sciences will be intimately bound with mathematics. Application of statistics has already contributed much to a systematic study of agricultural problems not to mention its many-sided applications to industrial problems.

I wish your conference a very successful and useful session.

* * *

From HIS EXCELLENCY C. RAJAGOPALACHARI, Government House, New Delhi.

It is the sacred duty of the Mathematicians of Free India to achieve something worthy of our illustrious past in Mathematics. This is my message to the Indian Mathematical Society at their conference in December 1947. I am sure India has a great future in Science and particularly in Mathematics, if only we strive hard.

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From HIS EXCELLENCY LT.-GENERAL SIR ARCHIBALD NYE, G.C.I.E., K.C.B., K.B.E., M.C., Governor of Madras, Madras.

As Chancellor of the Andhra University it is to me a great pleasure to be able to send this message of good wishes to the Indian Mathematical Conference on the occasion of its fifteenth session, which is now being held under the auspices of the Andhra University. Mathematics is an ancient Science which however keeps ever young and developing. India has played a distinguished part from the most ancient times to the present day in the progress of this branch of knowledge. The world expects India to continue to play the same big part in the future. Let me wish the Conference all success.

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From THE HON'BLE MR. T. S. AVINASALINGAM, Minister for Education, Government of Madras, Madras.

Let me thank you for your kind invitation for the 15th Biennial Conference of the Indian Mathematical Society. This great country is in the threshold of a new era. It has been given to us to be architects of a great and glorious future for this great country. I hope your Conference will give the members of your Association the vision and the enthusiasm necessary for this purpose.

I wish the Conference all success.

* * *

From SIR SHANTI SWARUP BHATNAGAR, KT., K.B., F.R.S., D.SC., Director, Council of Scientific and Industrial Research; Secretary, Ministry of Education, Government of India, New Delhi.

I take great pleasure in being associated with your 15th biennial Conference even though it is through the medium of a message.

Mathematical research is bound up so much with progress in fundamental research that your deliberations in the Conference and the lead you give to the research workers throughout India have a very special significance.

Perhaps the Indian Mathematical Society is unique amongst the learned societies in India in that it is giving equal attention to the teaching of mathematics. Your quarterly journal, "The Mathematics Student" deserves to be better known. With the present trend of developments it is necessary that attention is focussed on evolving better methods of teaching mathematics. It is a fact which cannot be easily ignored that quite a large proportion of students not only do not find any interest in mathematics but are actually scared away by it. Mathematics, perhaps, has been responsible for blasting more promising careers than any other single subject. We have so long taken for granted that these unfortunate people have no aptitude for mathematics. While this may be so to a certain extent, it is nevertheless a fact that no serious effort has so far been made to reorientate the approach to the subject so as to create and hold the interest of the pupil and to excite his curiosity. The great progress made in the Physical Sciences has been in no small measure due to the curiosity that a study of the subject has aroused in the young minds. It is time now that our mathematical experts looked into this most important aspect of an educational problem.

I wish your Conference all success.

* * *

From RAJA SIR ANNAMALAI CHETTIAR of Chettinad, the Palace, Chettinad.

It is gratifying to know that the Indian Mathematical Society are holding their fifteenth biennial conference at Waltair this year.

You ask me for a message. I do not think I need say more than wish the Conference all success from every point of view. The labours of the Indian Mathematical Society are something in which all men are interested. We, Indians, are proud of our mathematicians, both past and present, and I wholeheartedly join in the tribute which the Andhra University will pay them and wish the Conference all success.

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From Dr. R. M. ALAGAPPA CHETTIAR, M.A., D.LITT., LL.D., Barrister-at-Law, New Delhi.

I am most grateful to you for your kind letter of the 16th November 1947 asking me to send a message to the Indian Mathematical Conference to be held on the 22nd, 23rd and 24th of December. It is a rare privilege to be called to send out a message to a gathering attended by distinguished intellectuals. The value of such Conferences cannot be overrated by any Society. This high level pursuit of truth is a tribute to the minds that are engaged in it. A singular devotion to Science and Scientific Truth is a prerequisite of an advanced society. Disinterested intellectual curiosity is the very life-blood of any great civilisation. India is rightly proud of her ancient heritage and it is just as well that conferences of your nature are held in our country to remind us of our duty to the Kingdom of knowledge—a Kingdom which shall never perish. When the world is under a mad pursuit of worldly wealth and scientific knowledge only for destructive purposes, it augurs well for our country that a band of distinguished men is seriously engaged in the quest for truth just for its very delight. May such Conferences be a regular feature in our country is my earnest prayer. I wish your Conference all Godspeed.

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From SIR R. P. PARANJPYE, M.A., D.SC., Poona (Telegram).

Best wishes successful session Mathematical Conference.

Presidential Address

BY

PROF. DR. M. RAZIUDDIN SIDDIQI, PH.D., HON.D.SC., F.N.I., F.A.SC.,

Director, Research Institute, Osmania University, and President, Indian Mathematical Society.

I am deeply conscious of the great honour that has been bestowed upon me by my fellow mathematicians, and it is my first duty on this occasion to offer my grateful thanks to them for having elected me President of the Society for the ensuing term.

I have had the good fortune, during the last fifteen years, to be associated with the work of the Society in various capacities, and beginning with the Jubilee Session at Bombay, I have attended many conferences held at various places like Delhi, Lucknow, Hyderabad and Aligarh. But this is the first gathering of the mathematicians of Free India, and I shall always cherish with pride the memory of having presided over the deliberations of this Session.

The whole civilized world acknowledges the debt of gratitude it owes to this great country of ours for its early pioneering efforts in systematic mathematics. The discovery of the numerals and of the positional notation in arithmetic are two of the most fundamental and far-reaching contributions made by this ancient land to the evolution of civilised society. It is hardly possible to calculate the innumerable benefits that the human race has derived by the use of the mathematical discoveries of our ancestors in this country.

Now that the motherland breathes free air again, it is to be hoped that her sons and daughters will bring fresh glory to her name by achieving great things in every sphere of civilised activity, and particularly in the branch of knowledge in which Bhaskara, Aryabhatta and Srinivasa Ramanujan have made epoch-making discoveries.

From this brief reference to the past, let us turn to the present and the future. It cannot be denied that compared to the size and the population of our great country, the number of mathematicians is much too small, and the number of those actively engaged in research is smaller still. I had the opportunity, last year, of attending the summer meeting of the American Mathematical Society under the auspices of the Cornell University. Although many meetings of the Society are held every year, and regional meetings are also held at various centres, the Cornell meeting was attended by no fewer than four hundred mathematicians ! Of course, quantity is not necessarily and not always a true indication of quality, but it should be

admitted that an appreciable number of good, if not outstanding, mathematicians cannot exist if the total number itself is negligibly small.

The other day, when I requested a well-known mathematician to organise a symposium for this Conference, he replied back saying that it is no use having a symposium when there are hardly two people in the country working on the same subject.

It is not to our national interest to ignore the fact that all is not well with us in this respect, and that the position of mathematics in the country is far from satisfactory.

The causes for this state of affairs are not far to seek. The present system of education introduces too early and too narrow specialisation in the secondary school stage itself, and the young students naturally take up those subjects which enable them to pass the examinations easily, *i.e.*, the arts subjects, or subjects for which there is more or less inflated market value at present, *i.e.*, chemistry and biology. Hence it comes to pass that mathematics is being relegated to the background. Of the students who take it in the High School and Intermediate the majority do so not because they are interested in the subject itself but because they wish to take a degree in Engineering or some other technical subject.

Now there is no doubt that we are living in an age of specialisation, when almost all of us must be experts in some field in order to make a living. But as in other cases, so in this case also we have gone beyond the limit, and have neglected to envisage the fact that a society controlled wholly by specialists is not a wisely ordered society. Any one who visits a conference on scientific or technical subject would readily observe that no specialist speaks the language of the other specialist. Some author has defined the narrow specialisation of the present era as consisting of " knowing more and more about less and less". If this process is continued any further I am sure that such a specialisation would result in our knowing everything about nothing.

In order to discharge his duties as a citizen adequately a person must be able to grasp the complexities of life as a whole. The aim of education should therefore be to prepare an individual to become an expert both in some particular vocation or art and in the general art of the free man and the citizen. In other words, a useful citizen should not only be master of one subject but also jack of other basic subjects. Hence the importance of the movement for general and liberal education which is being revived in western countries.

In this revival of the general education, mathematics is regaining its former place as one of the most important and fundamental subjects. I

have found that in the American Universities a fairly advanced knowledge of mathematics is required from graduate students of the humanities and social studies.

In this competitive age when whole countries and nations are striving for supremacy over others in attaining power, it is but natural that every one should be on the look out for short cuts at acquiring technical skill without the necessary fundamental training. It is no wonder therefore that the aspect of science as knowledge is being thrust into the background by the aspect of science as the power of manipulating nature. Hence we observe the efforts made by our educationists to discard as much of the fundamental mathematics as possible, and to go straight ahead with the so-called practical subjects. What these people fail to appreciate, however, is that mathematics is woven inextricably into the structure of science and technology. What might be conveyed of physics, chemistry and biology without quantitative measurement and their mathematical manipulation, is not only not science but is definitely anti-scientific. It is nothing but the dilettantism of the Middle Ages.

In the social studies also, such as economics, sociology, anthropology and psychology frequent and increasing use is being made of what may be called college mathematics. Almost all students meet one or more of these fields either in the course of their higher studies or later in life, and hence should be prepared early with the simple mathematical techniques required for their pursuit.

As the Harvard educationists have pointed out recently, not only the college student, but a much larger and constantly growing section of the general population stands in need of mathematics. The complexities of organisation and technology in modern industry, in government, and in the national defence make increasing demands upon the mathematical equipment and skills of the ordinary participant and worker. Beyond this however, mathematics has an important intrinsic role in general education. It helps to build some of the skills and comprehensions that make the effective individual. Within the past fifty years mathematics and logic have been fused into a single structure, and the logical reasoning and deduction which are important features of mathematical procedure are required at every step in a rationally organised society.

General education throughout its history has included mathematics as one of its major components. It has lost none of its relevance in modern general education, though to it now must be added the enormous utility of mathematics in modern life. This is the opinion of some of the most eminent educationists of America, and it should provide serious food for thought to those educational authorities in our country who wish to abolish

from our schools all but the simplest arithmetic and mensuration which have an immediate market value. The catalogue of formulæ and rules of thumb which the students are expected to cram into their heads and reproduce during examinations cannot be expected to inspire them or give them a foretaste of and an abiding interest in higher mathematics. Is it any wonder, then, that so few students take up the subject in their college and university career? If it is desired that the number of students taking mathematics as optional in the Intermediate and higher stages should increase, and the mathematics should serve the purpose for which it is included in a system of general education in the schools, a drastic reform should be made both in the syllabus and the method of teaching in the secondary and the high school stages.

Let us now consider the mathematics that is taught to the undergraduate and graduate students in our colleges and universities. For obvious reasons, our college and university education was modelled upon the English system, and possesses consequently the virtues and defects of the parent organisation. In his Presidential Address to the London Mathematical Society, Professor Hardy has made a frank and searching analysis of the mathematics that is taught in the English universities and particularly in Cambridge, the Mecca of Mathematicians throughout the British Commonwealth. The same criticism can with equal justice be levelled against our own colleges. Even after taking their Honours or Master's Degrees, our graduates know very little of really modern mathematics, developed during the last one hundred years. Much valuable time and energy of the students is wasted in studying topics which have lost any importance that they once possessed. Another channel through which a good deal of wastage occurs is the custom which requires the student to solve artificial and twisted problems. Problem-solving is no doubt an essential feature of the training in mathematics, but the problems need not be of the head-splitting type indulged in by the examiners for the Tripos at one time, and condemned in no uncertain terms by Hardy and others.

If we keep in view these two facts, and revise our mathematics syllabus for the Intermediate and the B.A. accordingly, we can introduce a fair amount of modern topics during the last years of the Honours and the Master's Degrees. Our students would then have acquired a foretaste of mathematical research which they would want to continue with an active research career.

Turning now to research itself, we find a welcome sign that the authorities and governments are becoming research-minded, and are realizing the importance of research in national planning. But here again we come across a tendency to emphasize only one aspect of research, namely,

industrial' research, and to lose sight of the other part of the scheme, *i.e.* fundamental' research. The two can be considered as 'short-range, and 'long-range' schemes. No country can afford to ignore long-range plans if it wishes to develop properly, and to keep pace with others in the long run. Moreover, industrial research and its application for the betterment of society are entirely dependent upon fundamental research. It is necessary to emphasize the point that applied science cannot live on fundamental discoveries of the past only. It becomes stagnated and dies of inanition unless fresh discoveries are made. Even from the utilitarian point of view, therefore, it is obvious that fundamental research is not a luxury, but an absolute necessity. There is a well-known saying of the late J. J. Thomson which brings out this point very clearly. Leading a delegation of the Royal Society to the Lord President of the Council, and pleading for the claims of pure research which were in some danger of being swamped by those of applied research, J. J. Thomson asserted that:

> "Pure science is the seed of applied science, and to neglect pure science in a thing of this kind would be like spending a very large amount on manuring and ploughing the land, and then to omit the sowing of any kind."

As Mathematics is the basic pure science, it is essential for our progress in the long run that we should devote our resources to it in proportion to its importance, and should train a large number of our young mathematicians for careers of active research. This must be done not as a concession to an old and honoured subject, but as a measure of long-range planning, in the same spirit in which the National Physical and Chemical Laboratories are being organised, for ultimately the development of the physical sciences would depend upon the development of mathematics.

It was necessary to make this brief reference to the problem of reorganizing mathematics in the country as a part of the programme for national development which Free India must be considering now in all departments of collective activity.

It is customary in such conferences that the President should speak in his address about some topic in his science. I do not wish, however, to inflict on the general audience present on this occasion a technical account of the specialised topics in mathematics on which I have worked at one time or other. I have therefore chosen a topic which has a vital significance for every mathematician, and on which very little work is being done in this country. I mean the problem of the "Foundations of Mathematics".

The average student of mathematics is hardly aware that any such problem at all exists. From ancient times, it is commonly believed that mathematics is the most exact branch of knowledge, and although even the

greatest mathematician may sometimes err in his calculations, it is inconceivable that anything could be wrong with his principles and arguments. But this is what actually happened. The old masters were so busy raising the lofty and imposing structure of their subject that they seldom stopped to think whether the foundations on which they were building were secure. For instance they used infinite series quite freely without bothering to consider whether these series were convergent. At times, they even employed series which are now recognized to be divergent. Gauss and Cauchy were the first mathematicians to realize the insecure foundations on which mathematics was built, and they turned their attention to a critical examination and rigorous formation of the whole subject. The work was then carried on by Abel, Riemann and Weierstrass. Later on the work was taken over by Kronecker, Dedekind, Cantor, Poincare, Russel, Brouwer, Hilbert and a host of other writers, and has continued up to the present time.

This process of rigorous formulation led to the great and important movement in mathematics which started in the last quarter of the 19th century, and which is called the "Arithmetisation of Mathematics". The fundamental thesis of this movement is that intuitions based on our hazy notions of space and magnitude are inexact and often misleading, and that all mathematics should and could be derived, from the natural numbers. The process of counting is an exact one whereas measurement can only be carried out with a greater or less degree of approximation.

In accordance with this doctrine that mathematics must rest upon a purely arithmetical basis, the theory of irrational numbers so essential in analysis, was satisfactorily given by Weierstrass, Cantor and Dedekind. These researches in the theory of numbers led naturally to the theory of sets or aggregates, founded and developed by Cantor. The fundamental significance of Cantor's theory of sets is the establishment of the actual existence of the infinitely great number on the same basis as the finite number. To show what a revolution it means in our ideas, it is sufficient to recall a passage from Gauss' letter to Schumacher. Writes Gauss:—"I protest against the use of an infinite number as an actual one; this can never be allowed in Mathematics. The infinite is only a *facon de parler*, a figure of speech,........" This horror of the infinite, expressed by Gauss in 1831, was the motto of the mathematical world. Cantor challenged this assumption of Gauss, and gave the infinite an equal footing with the finite numbers. It took the mathematical world some time to accept these novel ideas, but when Weierstrass, Hermite, Poincare and Mittag-Leffler championed them, they soon gained recognition. Cantor went further, and brought about an extension in the scheme of numbers beyond the infinite, at which one cannot but look with wonder and reverence, and without which it is impossible to go a step forward in the development of the theory of sets,

and consequently of modern mathematics. These numbers are known to mathematicians as "Alefs" or "Transfinite Numbers"

The methods of the theory of sets have had a far-reaching influence both on pure mathematics and its applications. The relation between the theory of sets and the theory of functions has become so close that the latter cannot be thought of as existing without the former. The theory of sets has brought about an extension and differentiation in geometry which was inconceivable previously; it has created the "Theory of Dimensions", and "Analysis Situs" (Topology). Recently, the theory of sets has been applied to the Theory of Probability, and to Physics.

Apart from these applications which are important in themselves, one of the main problems of the theory of sets is to investigate methodically the fundamental ideas of mathematics, such as the ideas of number, order, function, infinity, limit and continuity. Bertrand Russell, writing about Cantor's mastery over the world of infinity, speaks thus: "Zeno was concerned with three problems. These are the problems of the infinitesimal, the infinite, and continuity. From his day to our own, the finest intellects of each generation in turn attacked these problems, but achieved broadly speaking nothing. Weierstrass, Dedekind and Cantor have completely solved them. This achievement is probably the greatest of which the age can boast".

The theory of sets contributed a great deal towards clearing the foundations of mathematics, but the theory itself was not founded on a secure basis as became increasingly evident to the mathematical world at the beginning of the present century. It was discovered that the idea of the set or aggregate as formulated by Cantor, leaves room for contradictions which are paradoxical. These paradoxes had at first a crushing effect; it seemed as if the whole structure of mathematics was going to collapse. Frege, who had spent years trying to put the mathematics of numbers on a sound logical basis, and who had just finished his lifework "Grundgesetze der Arithmetik", closes the second volume with the following acknowledgement: "A scientist can hardly encounter anything more undesirable than to have the foundation collapse just as the work is finished. I was put in this position by a letter from Mr. Bertrand Russell when the work was almost through the press." Russell had just communicated to Frege his famous paradox on the normal sets.

But during the years that have elapsed since then, much has been written on this subject from the mathematical as well as the philosophical standpoint. Apart from the fact that mathematicians have learnt to develop the theory of sets in such a way that the paradoxes disappear, these studies have thrown much light on many of the most fundamental problems of human knowledge.

The solution has been found in three ways by different schools, each of which is sharply distinguished from the other. The difference is not only methodical; it consists mainly in the whole mathematical outlook. The controversy between the three schools touches the most fundamental questions of mathematics, and it is with respect to these that we speak of a crisis in the foundations of mathematics.

These three schools consist of the Intuitionistic school led by Brouvwer and Weyl, the Logistic school of Russell and the Formalistic or Axiomatic school of Hilbert.

The basic principle of the intuitionists is their sharp distinction between the constructions and pure existence theorems. In every branch of mathematics there are proofs for the existence of certain mathematical objects, *e.g.*, numbers, functions and sets, which show this existence not by really constructing them, but by a certain non-constructive process, for instance by showing that the non-existence of the object in question would lead to a contradiction of other recognised theorems and principles. The intuitionist considers such non-constructive reasoning as illegitimate. For him existence in mathematics means constructibility.

The second main belief of the intuitionistic school is the rejection of the principle of "Excluded Middle", *i.e.*, of the principle that one of two opposites must hold. This principle, commonly known as *reductio ad absurdum*, was a very powerful tool in the classical mathematics of the 19th century; it was often employed to prove the truth of a proposition indirectly by disproving its opposite. The intuitionist objects to this principle, and explains that the law asserting that out of two opposites one must hold, is applicable only to finite sets. He finds no reason whatever for supposing that a principle which is adequate for the finite will continue to produce consistent results when applied to the infinite. For example, it is wrong to say that "Either there is a natural number which possesses a given property, or there is no such number". The intuitionist believes in a third possibility that from the very nature of the number, nothing can be concluded about the existence or non-existence of numbers with the said property. Thus even if we have proved that the assumption of the falsehood of a proposition is false, we have by no means established that the proposition is true.

Closely related to the question of the excluded middle is the axiom of the solubility of every mathematical problem. It has been a special impetus to mathematicians to believe that all their problems are soluble in one way or the other. The effort for centuries together to prove Fermat's theorems and many other theorems would never have continued had there existed any doubt about their solubility. The same cannot be said about any other

branch of knowledge. The intuitionists are convinced that the certainty about every mathematical problem being intrinsically capable of solution is by no means justified. In their opinon, besides the two cases of an affirmative or a negative answer to a question, there exists a third alternative, namely that of its insolubility.

The consequences of these intuitionistic ideas were terribly far-reaching. They meant that a large part of the most valuable possession of mathematics must be thrown overboard. The intuitionistic programme was therefore considered by a number of mathematicians to be suicidal. Even though recognising the pressing need for reform, they abstained from the extremely radical measures of the intuitionists. This reform has been taken up by the second, the so-called logistic school, whose foremost exponent is Bertrand Russell, and whose other representatives are Zermelo and Wittgenstein.

The starting point of Russell is the rejection of the non-predicative definition, *i.e.*, the process by which a certain representative of a general thought-complex is defined with the aid of the totality of all the representatives of that complex. Russell has been successful in showing that all the paradoxes arise out of the use of such a non-predicative definition. He therefore lays down the rule "that no totality can contain members which can be defined only through the totality itself." In other words, "whatever involves all of a collection must not be one of the collection". Russell calls this the "Vicious-circle Principle", and develops it into his famous theory of types which is the basis of the whole gigantic work, *Principia Mathematica.*

But unfortunately, this prohibition of the non-predicative definition was not free from undesirable effects. The structure of classical mathematics could not be built on this basis. A havoc was created of almost the same magnitude as that created by intuitionism. To save this desperate situation, Russell put forward a special axiom—the "Axiom of Reducibility". It says that a propositional function of arbitrary order is equivalent to an elementary propositional function. In other words, the circle of elementary functions is so comprehensive that no function of higher order can be formed which is not equivalent to one of the elementary functions.

The Reducibility Axiom differs from all other logical and mathematical axioms in that it is not tautological, that is, intrinsically necessary. It is even believed to be contradictory to the recognised axioms of logic, and it has not been accepted by the majority of mathematicians.

The third school of thought in mathematics is that founded and developed by Hilbert, and known as the axiomatic or formalistic school. The origin and early development of the axiomatic method lies in the domain of

geometry. The axiomatisation of geometry was brought about by Euclid, and later by Gauss and Lobatchewsky when they discovered the non-Euclidean geometry. The more general axiomatic treatment began with Pasch's lectures on the New Geometry in 1882, and reached its culmination in Hilbert's foundations of geometry in 1899. In this evolution from Euclid to Hilbert, the axioms have changed their character completely. They have been dethroned from their privileged position as self-evident truths, and have been placed upon the same footing as other propositions. They are chosen as the starting point only because they are more suitable for the building up of the theory.

In the year 1922, Hilbert began a series of articles on the Foundations of Mathematics. The most important characteristic of this new development is its tendency to separate the main body of mathematics on the one hand, and the foundations of mathematics on the other, into two different sciences. To distinguish it from the other, Hilbert calls the latter "Meta-mathematics". Contrary to the ordinary mathematics which is concerned with the formal building up of ideas, Meta-mathematics deals with the propositions and inferences of the formal mathematics. Just as the physicist investigates his apparatus, and just as the philosopher must criticise his own rational faculty, similarly it is necessary for the mathematician to secure his theoretical structure by a criticism of the principles and proofs on which it is founded.

The fundamental questions in the domain of Meta-mathematics are those of the Independence, Completeness and Consistency of the axioms.

First of all, we remark that the problem of selecting suitable axioms for a science in order to be able to deduce all the remaining theorems from them does not appear to be a definite one. It remains completely arbitrary as to which and how many theorems we choose as axioms. What limitation is then to be set for the number and character of the axioms ? This question is answered by the fundamental principle that none of the axioms should be superfluous or, in other words, that it should not be possible to prove any axiom with the help of some or all the remaining ones. An axiom possessing this property is said to be independent of all the other axioms. It is therefore necessary to prove that every axiom of the system is independent. This is done by constructing a pseudo-system or "model" in which on account of a different significance given to the primitive concepts, the axiom in question does not hold, while all the other axioms remain true. This pseudo-system is then either recognised to be free from contradiction or is proved to be so. From this it is concluded that the axiom in question is independent of the other axioms, for if it could be deduced from the others, then it should also hold in the pseudo-system.

From this application of the independence problem, we see its close connection with the problem of consistency. For, as we have seen above, to

prove a system of axioms independent is to prove that a certain pseudo-system is free from contradiction, *e.g.*, by the actual realisation of that pseudo-system. The parallel axiom of Euclidean geometry was finally proved to be independent only when Felix Klein was successful in establishing a correspondence between the points and lines of a non-Euclidian geometry with certain objects of the Euclidian geometry in such a way that the consistency of the Euclidian geometry implied that of the pseudo-geometry.

Along with independence, there is a second and more important property which is ascribed to a system of axioms. This is the property of completeness. The completeness of a system of axioms implies that it must contain the whole theory to be developed in such a way that every question which can be asked in terms of the basic ideas must be capable of being answered in one way or the other by pure deduction from the axioms. Such a characteristic of the system would mean that no further axioms can be added to it, and the system is therefore "complete".

The problem of the independence and completeness of axioms has vital significance because it is through these that the axiomatic method can claim that economy of thought which, according to Ernst Mach, is the ultimate goal of all science.

But the most important, and also the most difficult, problem for the mathematician is the proof of the consistency of his axioms, that is the proof that the axioms are individually and mutually free from contradictions.

When the Greek mathematicians, such as Euclid, tried to develop geometry on the basis of axioms, the problem of freedom from contradiction must have possessed no significance for them. The primitive ideas about which the axioms asserted something were taken over from life, and the axioms were considered to be partly analytic judgments and partly as facts of immediate experience or intuitions. Considered in this light, they did not appear to need any formulation, and being evident they could not lead to any contradictions. Such a consideration appears to us to-day to be quite unsatisfactory, partly because our belief in the infallibility of our experience and intuition has been shaken by certain mathematical facts, like the non-Euclidean geometry, or the continuous non-differentiable functions. Another cause which has caused this dissatisfaction is that we have no right to trust our immediate intuition considering the development which has taken place in connection with the choice of axioms out of a large number of propositions. If we want to select a number of theorems from any science, *e.g.*, Geometry, or Algebra, or Theory of Sets, in order to consider these selected theorems as a system of axioms, then as stated above, we have to take care that the system is independent and as complete as possible. We are still at liberty to choose any of the theorems as axioms. In practice, the axioms

are selected more or less in the following manner: The simplest theorems about which there is a certainty that all other theorems can be deduced from them are investigated in the light of the general and simple propositions that they contain. If these propositions, apart from being evident, or from agreeing with experience and intuition, are sufficient in order that the primary theorems can be deduced from them, then these propositions are taken to be axioms. These criteria of generality and simplicity have evidently a relative value, and it is therefore not surprising that the axiomatisation of a science is a matter of time and taste, and can never be unique.

Having once selected a set of independent and complete axioms, the question arises whether they are individually free from contradiction and mutually consistent. It is not sufficient to show that each individual axiom is free from contradictions, for it is quite possible that in a system each individual axiom may in itself be free from contradiction, while they may not be mutually consistent, and thus may lead collectively to contradictions.

For the mathematician this question of the proof of freedom from contradiction is a very acute one, and it takes a predominant part in the literature on the Foundations of Mathematics. We are again indebted to Hilbert for a satisfactory solution of this problem in most of the cases. This solution is obtained by arranging the definitions and axioms of the science in question (*e.g.*, Geometry) in one to one correspondence with those of another science (*e.g.*, Arithmetic) which has already been proved to be free from contradictions. The arrangement is such that every theorem of Geometry is transformed into a corresponding theorem of Arithmetic. Now if the axioms of Geometry contain a contradiction, *i.e.*, if they lead to a contradictory geometrical theorem, then its translation into the language of Arithmetic will give us a contradictory arithmetical theorem, which is impossible according to our assumption that Arithmetic has already been proved to be free from contradictions. Hence it is established that the axioms of the science in question are consistent.

This is a rough sketch of the new branch of mathematics, which has originated and developed in our own generation. It has been mentioned here with the hope that it would attract some of our mathematicians. Our countrymen have a reputation abroad for possessing an extraordinary flair for abstract philosophical speculations and reasoning. The kind of problems presented by the Foundations of Mathematics should therefore prove congenial to them, and I hope that we may see some further developments in the subject in the near future.

JAI HIND

Civic Address of Welcome Presented to the Delegation of The Indian Mathematical Society

(15TH BIENNIAL CONFERENCE)

LADIES AND GENTLEMEN,

We have great pleasure in extending to you a hearty welcome to our beautiful little City. We are persons preoccupied with the work relating to the mere terrain, and we look with wonder at the magnitude of scientific pursuit and the vistas of mathematical enquiry that are carried on by you.

Our City nestles in the hollow of a semi-circular chain of hills, the sea-front on the east forming the diameter with its north and south points projecting into the sea. Behind, in the west, is the harbour area, bounded by the hills. We have plans to develop the City all along the sea-front and on the hills where they meet the sea and behind and round the harbour. And as the water area of the harbour is increased to develop into an Inland Sea, as it were, the City will have a second arc-shaped sea-front. The present scarcity of water will shortly be a thing of the past and the present congestion too will soon cease to be and when we welcome you again on a future occasion, we believe that our vision will have been greatly fulfilled.

We know that you have lifted the name of India high in the world of scientific thought. We are proud of you all; you will, however, pardon us if we make a special mention of the name of Prof. C. V. Raman; for it was on this singing seashore and on these green hills that he roamed, when young, looking into the distant future; it was here that he, the bright son of a bright father, had his early schooling. Thus, when we think of scientific pursuit, we think of Raman and when we think of him, we feel a good deal of pardonable pride.

Whether the day-to-day world ever thought of the scientist and the pure mathematician some years ago as having a bearing on life, now at least after Hiroshima, it is otherwise. The world daily thinks of the scientist and feels it is a babe in the hands of Science. But will you be a plaything in the armoury of Visvamitra, timing your foot to the Dance of Shiva, or will you be a Kamadhenu of Vasishta spreading peace and plenty to all, in all lands and climes in an equal measure?

These are the thoughts that are passing our mind as we are welcoming you on the occasion of your first Conference after the attainment of Independence of this great and ancient land. We, however, have firm faith in the future and with that faith, we extend to you, our these sincerest greetings.

We remain,
Ladies and Gentlemen,
CHAIRMAN AND MEMBERS OF MUNICIPAL COUNCIL.

VIZAGAPATAM,
Dated 22-12-'47.

Papers Presented to The Fifteenth Biennial Conference, Waltair

G. A. MILLER, Illinois: Babylonian logarithms.

V. GANAPATI IYER, Annamalainagar: The space of integral functions.

V. S. KRISHNAN, Madras: Extensions of partially ordered sets.

A. C. CHOUDHURY, Calcutta: On quasigroups.

REV. FATHER C. RACINE, Madras: Determinants over a non-commutative field.

V. RAMASWAMI, Waltair: On the asymptotic distribution function of positive integers $< x$ and free of prime divisors $> x^{\epsilon}$.

K. SAMBASIVA RAO, Waltair: On the distribution of a class of primes.

C. S. VENKATARAMAN, Trichur: On the identical equations of multiplicative functions of two arguments.

D. R. KAPREKAR, Devlali: Products of $(9)_n{}^2$ with peaked step numbers.

D. B. LAHIRI, Calcutta: Certain identities connected with the number 5.

K. S. KRISHNAN, Delhi: On certain infinite series and equivalent integrals.

MAX DEHN, Black Mountain, U.S.A.: On the approximation of a function by a power series.

T. VIJAYARAGHAVAN, Waltair: On the approximation of a function by a power series.

REV. FATHER RACINE, Madras:
(i) On Frullani's integral.
(ii) On the notion of a convergent integral.

N. G. SHABDE, Nagpur: On a definite integral involving Legendre and Bessel functions.

S. M. SHAH, Aligarh: A note on uniqueness sets for entire functions.

U. C. SHARMA, Khurja: Some properties of a function considered by Ramanujan.

K. G. RAMANATHAN, Hyderabad: On the algebraic differential equations satisfied by some functions.

E. H. NEVILLE, Sonning-on-Thames, England: The bicircular generation of a conic.

R. S. MISRA, Lucknow, and RAM BEHARI, Delhi: Strazzeri's formula in rectilinear congruences and its applications.

M. VENKATARAMAN, Annamalainagar:
(i) The Langley chain.
(ii) On a principle of transference from the geometry of the plane to the geometry of lines in S_3.

A. NARASINGA RAO, Waltair: On a certain metric connected with line elements in inversive geometry.

K. RANGASWAMI, Annamalainagar: A note on the Hart Cubic.

A. A. KRISHNASWAMI AYYANGAR, Mysore: Some theorems on moment statistics.

S. M. KERAWALA, Calcutta:
(i) 6×6 latin squares.
(ii) Self-conjugate latin squares of prime degree.

B. S. SAXENA, Aligarh: On the porocity constant.

A. BHATTACHARYA, Calcutta: Theory of estimation.

B. R. SETH, Delhi: Finite longitudinal vibrations.

K. NAGABHUSHANAM, Waltair:
(i) An extension of a result of Wintner.
(ii) A generalization of the virial theorem.

D. ARKASOMAYAJI, Bhimavaram: Bhaskara's contribution to the evolution of astronomy in India.

T. V. VISWANATHAN, Madras: Distribution of the ratio of ranges in samples from a rectangular population.

THE MATHEMATICAL EXHIBITION

The Mathematical Exhibition arranged in connection with the Conference covered two spacious halls in the Vikram Deo College of Science.

All along the 4 sides of one of the halls were arranged Portraits of Eminent Mathematicians, Astronomers and Physicists from the Scripta Mathematica Portrait Gallery, each with a folder giving a description of the life and work of the person. High up on the walls were the values of e and π to about 400 and 707 places of decimals respectively. In the intervals between the portraits were various " Brain Teasers " and " Do you know " series of leaflets each containing an intriguing problem which could be solved without much knowledge of mathematics or interesting items of information, connected with mathematics. The central portion of the first hall was earmarked for " Mathematical Recreations ". There were a number of volunteers in this central portion, each in charge of one Problem or Game, explaining it or playing it with the visitors. The recreations thus arranged included among others—Bachet's weight problem, Battle of Numbers, Think of a number problems, Magic windows to show the number thought of, Jealous husbands passage, Oil monger's measuring problem, the eight queens problem, Solitaire, Nim and its generalizations, Tower of Hanoi, Hamilton's Dodecahedron problem, the 15 Puzzle, and Mac Mohan's coloured cube problem.

Among the exhibits in the second hall the place of honour was given to a life-size bust of Sir C. V. Raman with descriptive material all around listing his important contributions to Physics with their dates. There were a large number of Geometrical models in the same room—wooden and wire models of surfaces of the second degree, and their systems of circular sections, triply orthogonal confocals, helicoid, etc., surfaces of constant curvature, etc., kindly lent by the Annamalai University. There were also a number of interesting plane drawings kindly lent by Mrs. A. V. N. College, Vizagapatam. Among items of Astronomical interest were charts illustrating the relative sizes of the various planets, the age of the Earth, and various types of perpetual calendars.

Several charts illustrating optical illusions were on exhibition in this hall and attracted attention. These included parallel lines which did not look parallel, equal areas which belied their equality, straight lines which looked broken, sectors marked black and white which appeared to show colours when rotated and so on.

Among items of statistical interest were an electrical calculating machine which performed division automatically, and a collapsible cardboard model

of the normal surface both of which were kindly brought and exhibited by Dr. U. Sivaraman Nair of Trivandrum.

A very interesting set of drawings of great beauty were presented by the pupils of the 5th and 6th forms of the Board High School, Usilampatti, and were exhibited and explained by Sri. M. Parameswara Iyer of the same school under whose guidance and inspiration they had been prepared. These included the following items and formed a strong centre of attraction.

Six diagrams illustrating the triangle and its circles, a series of ten graded charts illustrating Feuerbach's Theorem prepared by P. Muthuramalingam of the 6th Form, more detailed charts of the same done in colours, figures representing properties of Quadrilaterals, geometrical illustrations of algebraic identities, etc. For work done by high school students, the diagrams were remarkably good.

A large part of the material relating to geometrical models in wood and wire, Brain Teasers, values of e and π and also those relating to recreations, were kindly lent by the Annamalai University.

For the convenience of those visiting the Exhibition a small cafetaria was arranged where biscuits and coffee were made available free to visitors to the Exhibition.

DISCUSSION ON THE TEACHING OF MATHEMATICS

Tuesday, the 23rd December 1947

Dr. M. Raziuddin Siddiqi, President of the Society, took the chair.

Prof. F. W. Levi (Calcutta) initiated the discussion.

Prof. T. Vijayaraghavan (Waltair) raised the question whether or not there was need for a degree higher than M.Sc. and lower than D.Sc. for those who wanted a very good acquaintance with higher mathematics, but who had no aptitude for research.

Rev. Father Racine (Madras) felt that mathematics, as taught at present, was absolutely uninteresting because the theories and problems discussed were old and had no living interest. Properties of particular curves, for example, were things of interest to mathematicians of the 17th and 18th centuries, but not at the present day. The teaching of mathematics has to be modernized. He remarked that in some French Universities they were teaching point set Topology, the theory of measure and the Lebesgue integral to students corresponding to those in our honours classes.

Sir C. V. Raman (Bangalore) said there were really two problems (i) to raise the standard of mathematics in Universities, and (ii) to broaden the basis of mathematics so that a large number of persons may receive mathematical training. Mathematics, he said, must be made more interesting, more living and more practical by work in a mathematical laboratory, where the student would learn to calculate and handle numerical data. He himself had learnt a good deal by actually sitting down and working out the Fourier Analysis of a weather chart. He said he had sometimes gone to bed with a copy of Hilbert and Cohn Vossen's *Anschaulische Geometrie.* As regards the second point he said that mathematics was a tool and the needs of Physicists, Chemists, etc., were different. There must be freedom to study, each in his own way and take an examination or not as he may decide.

Prof. S. Mahadevan (Saidapet) wanted an improvement in the technique of teaching. The young student had to be made to appreciate why he should learn mathematics. The teaching of mathematics had become mechanical and standardized, and nothing had been done to improve it. He suggested the appointment of a committee to suggest improved methods of teaching mathematics.

Prof. Levi (Calcutta) said that experiments in teaching even at the post-graduate stage were hampered by the examination system. Nevertheless they should be encouraged. Tutorial classes when properly managed could be much of a success.

Prof. A. Narasinga Rao (Waltair) said that an aversion to mathematics was sometimes developed at an early stage in the case of some pupils because of faulty methods of teaching. The remedy lay in the employment of a better equipped and better type of teacher (which meant also better pay). He heartily supported the idea of training in a mathematical laboratory as a corrective for many of the defects of present-day teaching. Such laboratory practice should not be reduced to a mere routine but must seek to develop initiative and should be run on heuristic lines. If the teaching was lamentable, our examination system was atrocious inasmuch as the percentage of passes even where a large number of candidates was involved varied widely from year to year.

Sir K. S. Krishnan (Delhi) said that the improvement of the teaching of mathematics was the main thing, and so long as we attended to it, the examinations might be left to themselves.

Sri. M. Parameswara Iyer (Usilampatti) said that the teaching of mathematics in the lower classes must aim at promoting habits of neatness and accuracy and that this could be done by attaching more importance to geometrical drawing. In order to promote quickness in oral work, he was in favour of the revival of teaching the "ensuvadi" such as was common in village schools.

Dr. Miss Bina Ghosh (Delhi) thought that if the teaching of mathematics was to be improved we should begin at the primary stage, and teach the subject in a practical manner so as to be related to the needs of everyday life. The pupil should see how useful it was. Arithmetic, algebra and geometry should not be taught in water-tight compartments. For example, the idea of ratio could be introduced through arithmetic, algebra and geometry.

Prof. N. M. Basu (Aligarh) claimed that he had taught mathematics for over 30 years but his own experience after devising various schemes for the improvement of teaching was that they did not work as planned, especially in the case of students who were unfit for the course and would not respond. There was no means of enforcing the carrying out of the programme. It almost looked as if we required two committees, one for planning courses and another for enforcing them.

Sir C. V. Raman: Let us try and do something at least in the Universities. Let us improve ourselves before we seek to improve others.

Prof. A. N. Singh (Lucknow) found that text-books in mathematics for high schools were often very bulky, consisting for instance of three separate volumes on (i) Arithmetic, (ii) Algebra and (iii) Geometry with

a total of 1,400 pages. Text-books, he said, should be good and short, and no book should exceed 200 pages.

Prof. Ram Behari (Delhi) remarked that the aim of teaching mathematics at different stages should differ. Thus at the school stages the stress should be on the utilitarian aspect and not on rigour. At the Inter and B.A. (Pass) stages even when non-rigorous proofs of theorems were given, the theorems should be *stated* rigorously, and the shortcomings in such proofs, should be pointed out, while at the B.A. (Hons.) and M.A. stages proofs should be quite rigorous. He pleaded for the teaching of mathematics in its historical setting, and for emphasis in its practical aspect at the earlier stages to make the subject more interesting and to impress its importance and usefulness. Students should be encouraged to frame questions and answer them themselves.

Prof. Gunjikar complained that Teachers of Mathematics, being generally specialists, often forget the needs of the common man, who may not be very mathematically minded, but only think of the students who are specially gifted in the subject. The needs of the two types of students are different and we must devise effective courses of study, methods of teaching and styles of examination papers to suit their different requirements and interests. Similarly the needs of students of science and technology, who are not generally interested in the rigour of pure mathematics, but have to apply the subject as an instrument, must not be forgotten. Teaching of mathematics has to be simplified for them to enable them to handle the subject with effectiveness and confidence, within the limitations imposed by their requirements.

Sri. S. Rghavachari (Student Volunteer) said that the teaching of mathematics should be made more related to practical life, and its problem. Only thus could we hope to secure greater interest in the subject. Propositions in the Theory of Numbers and in Group Theory will not appear difficult to students if illustrated by means of exhibits such as those found in the Mathematical Exhibition. He thought that there should be prize problems at various levels to be competed for by students.

Winding up the discussion, Prof. Siddiqi (Hyderabad) remarked that there was unanimity of opinion on one point, namely, the need for the revision of the present syllabuses and methods of teaching mathematics at all stages—the school, college and the university. The problem is not confined to one locality or even one country. Before the World War I, the International Congress of Mathematicians had appointed a commission to enquire into the whole problem. Many national committees were formed, and some of them had even published preliminary reports of work done in their respective countries. But due to the intervention of the war the

work was left incomplete. It is high time that the work is resumed and a comprehensive survey of the present position is made so that we may be able to lay down a definite programme of future reforms. The Indian Mathematical Society should approach the Central Government in this connection and request to be provided with the necessary authority and facilities and funds to do the work.

26 JAN 1950
MADRAS

FIFTEENTH CONFERENCE

BUSINESS MEETING OF THE SOCIETY

Tuesday, 23rd December 1947

Dr. M. R. Siddiqi, President of the Society, took the chair.

A condolence resolution was moved and passed, all the members standing, expressing the Society's sense of loss at the death of the following members in the period intervening between the previous Conference and the present one:

Prof. E. B. Ross
Prof. G. H. Hardy
Prof. A. C. Wilkinson
Prof. S. C. Dhar.

Dr. A. Narasinga Rao then made a statement about the financial position of the Society in which he drew attention to the phenomenal rise in the cost of printing while the income of the Society remained practically stationary. This was one of the reasons why the Society had been compelled to issue double numbers of the *Journal* and the *Student*. For a country of the size of India, the membership of the Society was rather poor and he invited the co-operation of those present in increasing the membership.

Dr. T. Vijayaraghavan, Secretary of the Society, spoke on the need for a thorough examination of the syllabuses and the methods for the various courses of teaching mathematics in this country, and suggested the appointment of a Committee to issue an exhaustive questionnaire covering all aspects and make its recommendations after considering the replies. After a considerable discussion, it was resolved that the following Committee be appointed for the purpose and that everything which appertains to the teaching of Mathematics should come within its purview:

Prof. M. R. Siddiqi (Hyderabad), Sir K. S. Krishnan (Delhi), Miss Bina Ghosh (Delhi), Prof. Ram Behari (Delhi), Prof. S. Mahadevan (Saidapet), Prof. Gunjikar (Bombay), Prof. A. N. Singh (Lucknow), Prof. B. N. Prasad (Allahabad), Prof. H. Subramania Iyer (Trivandrum), Prof. N. G. Shabde (Nagpur), Prof. N. R. Sen (Calcutta), Prof. A. C. Bannerji (Allahabad), Prof. N. M. Basu (Dacca), and Prof. T. Vijayaraghavan (Waltair), Convener. The Committee was requested to submit its final recommendations by September 1948.

The question of revising the constitution of the Society was raised by some members. It was felt, however, that in this matter the initiative might well come from the Managing Committee of the Society, and the matter was accordingly dropped.

SYMPOSIUM ON METHODS OF TESTING HYPOTHESES

Wednesday, 24th December 1947

Chairman : DR. U. SIVARAMAN NAIR, TRIVANDRUM

Summary of Chairman's Inaugural Speech on Tests of Statistical Hypotheses

There is a feeling among high-browed mathematicians that Statistics is anything but mathematics—that it may be ranked with Economics, Sociology or even Psychology. Such a feeling is unfortunate and I am all the more grateful to the Indian Mathematical Society for organising this symposium on Statistical Tests of Hypotheses.

I may be pardoned for starting this symposium with the explanation of a few preliminary concepts that are fundamental in theoretical statistics. An aggregate of elements—finite or infinite—constitutes a statistical population, denoted for short by π. An element of π can be specified by k variables $(x) = (x_1, x_2, .., x_k)$ which can be represented by a point P in a Euclidean k-space. The values of x for the i th individual will be denoted by

$$(x)_i = (x_{1i}, x_{2i}, .., x_{ki}).$$

The population is now equivalent to a set of points R—finite or infinite—in a k-space. Statistical theory relates to the study of this set. Clearly, the configuration of this set is known if a function $p\ (x_1, x_2, .., x_k, \theta_1, \theta_2, .., \theta_c)$ denoted for short by $p\ [(x), (\theta)]$ can be determined such that its integral over any region Γ belonging to R gives the number of points in Γ. There is no loss of generality if it be assumed that the integral of $p\ [(x), (\theta)]$ over the entire set R is unity. With this convention, $p\ [(x), (\theta)]$—called the probability or density function—uniquely determines π and hence is equivalent to it.

Statistical theory starts with the density function. Assuming its functional form to be known, the constants θ have to be determined. The material available for this is a known finite subset E of R — $[(x)_i, i = 1, 2, 3, .., n]$ — and this subset is called the sample. It is convenient to take the n values in the sample to be independent in the probability sense so that the probability of getting the sample is

$$p\,(\text{E}) = \prod_1^n p\ [(x)_i, (\theta)].$$

The problem of determining the θ's—called estimation—is based upon considerations which are in a sense purely arbitrary. The procedure is to take a property of π expressible in terms of θ's and to identify it with the

corresponding property in E also expressible in terms of the subset E. Thus if $f[(x)]$ is a function in (x) and

$$F[(\theta)] = \int_R f[(x)] \cdot p[(x), (\theta)] \, dx$$

and

$$F_0 = \sum_1^n f[(x)_i]/n$$

$$F[(\theta)] = F_0$$

is an equation to estimate θ. It will be seen that the usual methods of estimation are particular cases of the above principle. As an illustration, consider the method of maximum likelihood. Since

$$\int_R p[(x), (\theta)] \, dx = 1,$$

$$\int_R \frac{\partial p}{\partial \theta} \, dx = 0$$

or,

$$\int_R \left[\frac{\partial}{\partial \theta} \log p\right] \cdot p[(x), (\theta)] \, dx = 0$$

assuming that differentiation under the integral sign is valid and that the limits of integration are independent of θ. Setting $f[(x)]$ equal to the derivative of log p with respect to θ, $F(\theta)$ is zero.

Also

$$F_0 = \Sigma \frac{\partial}{\partial \theta} \log p \, [(x)_i, \theta] = \frac{\delta}{\delta \theta} \log p(E)$$

Thus the estimating equation is

$$\frac{\partial}{\partial \theta} \log p \, (E) = 0.$$

In spite of the arbitrariness in the different methods of estimation, 'good' estimates can be selected by imposing certain conditions on the estimates. Thus if T is an estimate of θ, one can use the conditions

(i) $\lim_{n \to \infty} T = \theta$ (consistence)

(ii) $E(T) = \int_R T \cdot p[(E)] \, dx = \theta$ (unbiassedness)

(iii) $E(T - \theta)^2$ is a minimum. (efficiency)

These three conditions will in general remove the arbitrariness of relevant estimates.

The second aspect of theoretical statistics relates to Tests of Hypotheses. Starting with the sample E, the problem is to conjecture the most likely population π from which E could have arisen. Since π is equivalent to $p[(x), (\theta)]$, one has to conjecture

(i) the mathematical form of p

(ii) the values of θ in p.

To simplify the problem, it will be assumed that the mathematical form of p is known. One has now to decide whether, based on E, θ can be equal to known specified values θ°. Sometimes, situations arise when one may be interested in only a few of the parameters. A statistical hypothesis is called 'simple' if all the c parameters θ are specified. The statistical hypothesis is 'composite' if only g parameters are specified leaving $(c - g)$ parameters free. $(c - g)$ is the number of degrees of freedom of the composite hypothesis. In either case, H° will be used to denote a specified hypothesis ascribing to some or all the θ's definite values θ° and H will denote any general hypothesis regarding the θ's.

The validity of H° will depend upon what happens when different samples E are available. If in a large succession of E's, H° seems plausible one has to accept H°. If on the other hand, it looks as if H° is highly unlikely based on the evidence of repeated trials, it has to be rejected. To formulate this procedure more precisely, consider the case when the population π relates only to one variable x so that the sample E reduces to n values $(x_1, x_2 .., x_n)$ of x. The sample can be represented by a point in a Euclidean n-space—called the sample space. Different samples will give different points and the aggregate of these points in the sample space will form a set W. Points lying in regions of crowding of W will correspond to samples occurring frequently while the region W $-$ A will contain rare samples. Clearly A will depend upon the θ's defining π. If H° assigns values θ° to θ, the corresponding region A will be denoted by A°. Thus a working rule to decide whether H° is true or not is the following—

Accept H° if the sample point E falls in A°

Reject H° if the sample point E falls in W $-$ A°.

Though an indication has been given to locate A°, certain other considerations have to be taken into account to fix A° uniquely. The principle involved in locating A° is based upon the fundamental rule governing conduct of life in any community. Consistent with present-day thought, one can assert that human action, at least in normal individuals, is the result of striking an equilibrium between the two opposing maxims—

(i) Do good as often as possible,

(ii) Do wrong as rarely as necessary.

Accepting H° $-$ *i.e.*, E ϵ A°—can be when (i) when H° is in fact true or (ii) when some other alternative H is true. Case (i) is certainly the right thing to do and is therefore 'good'. Case (ii) is doing 'wrong'. Thus

the two maxims reduce in mathematical language to

(i) $P\,[E \epsilon A^{\circ};\; H^{\circ}]$ is large

(ii) $P\,[E \epsilon A^{\circ};\; H]$ is small

where $P\,[X;Y]$ denotes the probability of X given Y.

It is usual to fix a high level —— $\alpha = 0{\cdot}95$ —— for the probability in (i) and determine A° subject to the conditions that (ii) has a minimum. Thus the equations to determine A° reduce to

(i) $\int_{A^{\circ}} p\,[E;\theta^{\circ}]\,dx = \alpha$

(ii) $\int_{A^{\circ}} p\,[E;\theta]\,dx$ has a minimum at $\theta = \theta^{\circ}$.

These are the two equations of Neyman and Pearson who gave the theory of tests of hypotheses its philosophic setting. A° is called the 'region of acceptance' of H° at 'confidence level' α. The region $W - A^{\circ}$ has been called the region of rejection or the 'critical region' of 'size' $1 - \alpha$. Further

$$\beta\,(\theta) = 1 - \int_{A^{\circ}} p\,(E;\theta)\,dx$$

is called the 'power of the test' based on the critical region $W - A^{\circ}$. It gives the probability of rejecting H° when some other alternative H is true. In terms of $\beta\,(\theta)$ the rule to determine A° reduces to

(i) $\beta\,(\theta) = 1 - \alpha$ at $\theta = \theta^{\circ}$

(ii) $\beta\,(\theta)$ has a maximum at $\theta = \theta^{\circ}$.

The problem so presented is clearly one of Calculus of Variations. Maximising $\beta\,(\theta)$ subject to the condition $\beta\,(\theta) = 1 - \alpha$ at $\theta = \theta^{\circ}$ reduces to finding the unconditional maximum of

$$\int_{W - A^{\circ}} [\,p\,(E;\theta) - kp\,(E;\theta^{\circ})]\,dx$$

In the present situation it will be seen that the common boundary of A° and $W - A^{\circ}$ is the set of points given by

$$p\,(E;\theta) - k.p\,(E,\theta^{\circ}) = 0.$$

Finally the constant k is determined so that the value of the integral of $p\,(E;\theta^{\circ})$ over A° is α. A further reduction is possible if $p\,(E;\theta)$ reduces to the form

$$p\,(E;\theta) = U\,(T,\theta).V(E),$$

where T denotes simple functions of E—called sufficient statistics for θ—and V (E) is a function of E not involving θ. In this case the above equation reduces to

$$U\,(T,\theta) - k.U\,(T,\theta^{\circ}) = 0$$

Thus the test can be reduced to one based on the sampling distribution of T only.

Even though the solution to the problem of tests of statistical hypotheses has been given a formal elegant solution, it is found that considerable difficulties have to be faced in their practical applications. For a discussion of these issues the original memoirs of Neyman and Pearson have to be consulted.

SUMMARY OF SPEECH

By Dr. Sundararama Sastry

(Reserve Bank, Bombay)

Neyman-Pearson criterion

Let x be a random variable and $f(x)\,dx$ its probability function.

Let $x_1, x_2, \ldots, x_n$ be n observed values.

The likelihood of the hypothesis H_0 being true is given by

$$p(x_1, x_2, \ldots, x_n/H_0);$$

The likelihood of hypothesis H_1 being true is given by

$$p(x_1, x_2, \ldots, x_n/H_1).$$

Let

$$\lambda = \frac{p(x_1, x_2, \ldots, x_n/H_0)}{p(x_1, x_2, \ldots, x_n/H_1)}$$

be so chosen as to make the probability of the error of the first kind have a pre-assigned value α. This α is called the size of the critical region. We call this critical region, the most powerful critical region if the above relation holds for any set of alternative Hypotheses $H_1, H_2, \ldots, H_r$. In such a case, for a fixed sample of size N the probability β of an error of the second kind is a single valued function of α, say $\beta_N(\alpha)$; and N is usually so fixed as to satisfy the condition $\beta_N(\alpha) \leqslant \beta$.

Thus, the most powerful test procedure for testing H_0 against H_1 can be stated as follows. Fix λ so as to make the probability of the error of the first kind $\leqslant \alpha$; and then fix up N so that the probability of the error of the second kind $\leqslant \beta$.

Sequential procedure

In the above procedure N remains constant for any particular problem. This may lead to a certain inefficiency because in certain cases, the hypothesis may be tested with a number of observations $< N$. For example, let ω_N be a most powerful critical region based on N observations, where ω_N is defined by the inequality $x_1^2 + x_2^2 + \ldots + x_n^2 \geqslant c$. Suppose for some

$n < N$, $x_1^2 + x_2^2 + \ldots + x_n^2 \geqslant c$. Thus the Null-hypothesis is disproved at the stage n; and the observations $N - n$ are unnecessary. Hence a procedure which controls the errors of the first and second kinds to exactly the same extent with a smaller number of observations is preferable to the above. The answer to this problem is supplied by the sequential probability ratio test.

Let the probability distribution of x under hypothesis H_0 be given by

$$f(x, \theta_0).$$

Similarly let the probability distribution of x under hypothesis H_1 be given by $f(x, \theta_1)$,

$$\lambda = \frac{f(x_1, \theta_0) \ldots f(x_n, \theta_0)}{f(x_1, \theta_1) \ldots f(x_n, \theta_1)}$$

$$\log \lambda = \log \frac{f(x_1, \theta_0)}{f(x_1, \theta_1)} + \ldots + \log \frac{f(x_n, \theta_0)}{f(x_n, \theta_1)}.$$

If $\lambda \geqslant A$ we reject the hypothesis H_0; if $\lambda < B$, we accept the hypothesis H_0. For $A \geqslant \lambda \geqslant B$ *we have the region of doubt.* It can be shown that $A \leqslant \frac{1-\beta}{\alpha}$ and $B \geqslant \frac{\beta}{1-\alpha}$. At each stage of the experiment, after the mth observation for each integral value of m, we compute λ. We stop the experiment when $\lambda > A$ or $\lambda < B$. Otherwise, we continue the experiment by taking an additional observation.

Thus the m dimensional space can be divided into 3 mutually exclusive regions R_m^0, R_m^1, and R_m. At the observation x_m, H_0 is accepted if the point falls in R_m^0 rejected if it falls in R_m^1, and the experiment is continued if it falls in R_m. The sequent probability test is an optimum procedure in the sense that it requires the minimum number of observations compared with any other test which controls the two kinds of errors to the same extent.

We say that two sequential tests S and S′ are of equal strength if $\alpha = \alpha'$ and $\beta = \beta'$. If $\alpha < \alpha'$ and $\beta \leqslant \beta'$ or if $\alpha \leqslant \alpha'$ and $\beta < \beta'$, we say that S is stronger than S′; If $\alpha > \alpha'$ and $\beta < \beta'$ or if $\alpha < \alpha'$ and $\beta > \beta'$ we should say that the strength of S is not comparable with that of S′.

Let $E_r(n/S)$ be the expected value of n if H_r is true under the hypothesis S. Then the efficiency of any sequential test S is measured by the ratio

$$\frac{\text{Max}\,[E_r(n/S''), E_1(n/S'')]}{\text{Max}\,[E_0(n/S), E_1(n/S)]},$$

when S^* is the optimum sequential test of strength equal to that of S.

The current test procedure can be considered as a particular case of sequential test taking that for all $m < N$, R_m^0 and R_m^1 are the expected observations of the m dimensional space μ_m and $R_m = \mu_m$.

For $m = N$,

$$R_N^1 = \omega_N,\ R_N^0 = \bar{\mu}_N \text{ and } R_N \text{ is that empty set.}$$
$$E_0(n) = E_1(n) = N.$$

Limitations of Sequential Method

(1) This method can be used with advantage in all empirical fields of investigation where the data have to be obtained from experiments which can conveniently be arranged in a time succession so that at each stage of the experiment the experimentor can freely decide whether the experiment be terminated or additional trial be performed. But where the experimentation requires a fairly long period this method is not applicable.

(2) So far, it is applied for testing a single hypothesis. Suppose K mutually exclusive and exhaustive hypotheses are involved, the problem is to accept one of them on the basis of the sample. Here is a wide field for development.

(3) Another important problem about which very little is known is that of sequential estimation.

Advantages

(1) While current tests cannot be carried out without finding the probability distributions of the statistics on which the test is based, there are no distribution problems in connection with sequential tests. The sequential probability ratio can be calculated from the data of the problem without solving any distribution problems. But these problems arise only if it is desired to find the probability distribution of the number of trials for reaching a decision.

(2) It has been proved that for the range of α and β from 0·01 to 0·05 the sequential process leads to an average saving of at least 47% in the necessary number of observations as compared with the current procedure.

SYMPOSIUM ON THE THEORY OF MEASURE

Lecture on "Haar-measure"

BY

F. W. LEVI

In a paper which has already become classical[1] A. Haar has developed a method by which in every locally compact continuous group G, a measure function, satisfying the conditions of Caratheodory and invariant for left side multiplication with group elements, can be introduced. The "Haar-measure" affords the integration of functions in the group-space G under conditions which correspond to those for Lebesgue integration in the Euclidean n-space.

If $f(x)$ is a continuous function in G, then

$$\int_G f(ux^{-1})f(vx^{-1})\,dx = \int_G f[u(xv)^{-1}]f[v(xv)^{-1}]\,dx = \int_G f(uv^{-1}x)f(x^{-1})\,dx$$

is a function $F(uv^{-1})$ of uv^{-1} and is symmetric in u and v. Let λ be an eigen-value of multiplicity n of the symmetric integral equation

$$\phi(x) = \lambda \int_G F(xy^{-1})\,\phi(y)\,dy$$

and $\phi_1, \ldots, \phi_n$ be an orthogonal basis for the solutions, then for every element a of G, there is

$$\phi_p(xa^{-1}) = c_{p,1}(a)\,\phi_1(x) + \ldots + c_{p,n}(a)\,\phi_n(x), \quad p = 1, \ldots, n.$$

The matrices $[c_{p,q}(a)]$ furnish an orthogonal representation of order n of the group G.

The method applied by Haar with its generalisation to arbitrary locally compact metric spaces affording a congruence relation was explained in the lecture. Owing to war and postwar-conditions more recent publications on the topic were unknown and unaccessible to the speaker. Of those, the pamphlet of *A. Weil*, L'integration dans les groupes topologiques et ses applications (originally published by Hermann & Co., Paris, lithoprinted by Edwards Brothers, Ann Arbor, Michigan), has to be quoted in the first line.

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[1] "Der Massbegriff in der Theorie der kontinuierlichen Gruppen," *Ann. of Maths.*, (II), 1933, vol. 34.

DETERMINANTS OVER A SKEW FIELD

BY

REV. FATHER RACINE, *Madras*

In this talk, is developed a theory published by J. Dieudonné in ***Bulletin** de la Société Mathématique de France*, tome LXXI, 1943, pp. 27–45.

The idea of Dieudonné is extremely simple; it consists in determining all the possible mappings of the group of the square non-singular matrices into an abelian group. Let $M_n(K)$ denote the group of the square non-singular matrices of rank n over the skew field K. Let K be the set of the non-zero elements of K. Let C_n and C respectively denote the commutator groups of $M_n(K)$ and of $K^{\times}$ (multiplicative group of K). Then

$$M_n(K)/C_n \cong K^{\times}/C$$

and the mapping of $M_n(K)$ into—or onto—$K^{\times}/C$ yields a generalisation of the notion of determinant.

The only difficulty of the question is relative to the point of determining whether for a given skew field the mapping is not "*into*" and in such a manner that the ony image of $M_n(K)$ is the neutral element (or unity). In a skew field where there is a norm—as in the case of the quaternions over the real numbers—the mapping is "*onto*". In more general cases, the question remains open.

REVIEWS

Lessons in Elementary Analysis

BY G. S. MAHAJANI

(Fourth Edition, 1949, pp. 334, Price Rs. 10, Aryabhushan Press, Poona)

The first edition of this book was published in 1929 and was designed to meet the needs of B.A. and B.Sc. students of Indian Universities. At each subsequent edition new matter was added on so that the present edition covers the greater part of the Honours course in South Indian Universities. In 12 chapters followed by an addendum, the author deals with the main topics in Analysis, *viz.*, the arithmetic continuum (with Dedekind's method slightly modified), the theory of limits including an excellent treatment of the exponential theorem, differentials and derivates, continuity, properties of continuous functions (nine theorems), Mean value theorems, their applications and generalizations, Taylor's theorem, Riemann integration, Mean value theorems of the Integral calculus (the second mean value theorem as per Edmund Landau), infinite integrals, uniform convergence of series and integrals—the parallellism between series and integrals being well brought out, inversion of operations, and Fourier series. The addendum contains notes on Gamma and Beta functions, Abel's theorem on power series, Taylor's expansion or several variables, Frullani's integrals, Stirlings' approximation, and more advanced theorems on Riemann integration.

It will be seen that all important topics are covered. The treatment is unusually clear and rigorous, difficult points in the proof being adequately explained. By means of illustrative examples the author brings out the essential points in the theorems which precede. Each chapter is followed by a set of examples some of them being intended to illuminate the theory and the others to stimulate further study. Every page bears evidence that the author has had long experience of teaching the subject and knows where exactly the learner is likely to need help and guidance. The chapter on the Mean value theorem of the differential calculus contains many interesting generalizations due to V. Ganapati Iyer, and C. T. Rajagopal each leading to Taylor's theorem and having the theorems of Cauchy, Lagrange and Rolle as particular cases, as also a second group including the theorems of Schwarz and the generalized result of Stieltjes.

There are a few misprints, such as, $\frac{1}{2}(l - l')$ for $\frac{1}{2}\,|\,l - l'\,|$ on p. 31, line 5; 1, 0, ∞ instead of 0, 1, ∞ in line 3 of p. 41; "one" for "none" in line 20, p. 90. Other slips noticed by the reviewer are: the word "Cauchy's" instead of "Lagrange's" on p. 132 in Queston 13; 7, 8, 9 and 10 instead of 5, 6, 7 and 8 on p. 201, line 9. On p. 146, second line from the bottom, it must have been stated that $|-x| < 1$, hence $\Sigma\, u_n$ is absolutely convergent and $u_n \to 0$. On p. 231, second line from the bottom, $-\frac{1}{2}$ must be added to the right side and the rest modified suitably. Having defined absolute convergence of $\int_0^\infty f(x)\,dx$ as the convergence of $\int_0^\infty |\,f(x)\,|\,dx$, the author should have shown that absolute convergence implies convergence.

Altogether an excellent book which can be warmly recommended to students who wish to get a real grasp of the subject.

M. LAKSHMANAMURTI.

Functional Analysis and Semi Groups

BY EINAR HILLE

(Vol. XXI, pp. xi+530, Price $ 7·50, American Mathematical Society Colloquim Publications)

This book is a valuable addition to the existing mathematical literature. It is divided into three parts.

The first part is a self-contained account of function theory in Banach spaces and Banach algebras. After introducing the fundamental notions about abstract spaces he gives an account of the theory of linear transformations in Banach spaces, then deals with such concepts as integrability, differentiability, analyticity of functions where the variables are elements in Banach spaces, and finally closes this part with an account of "analysis in Banach algebras". For the first time an account of the theory of Banach algebra—a subject of great interest particularly after the World War II—is made available to the English-knowing world.

The second part deals with analytic theory of semigroups. Attention on one-parameter family of groups is concentrated. Their analogy with exponential functions is brought out. The relationship between the resolvent of a linear operator and one parameter family of semi-groups by means of Laplace transforms is discussed. The author also studies Ergodic and Tauberian theories from the point of semi-groups.

The third part is a collection of various applications of the ideas in part II, to such topics as trigonometric series and integrals, summability problems, stochastic theory, Cauchy's problem in partial differential equations, etc.

The appendix on Banach algebra dealing with ideas not used in the theory of semi-groups is quite welcome and adds greater value to the book.

On the whole the book will be useful to any student of analysis interested in advanced work.

S. M.

Analytical Geometry of the Conic Sections

BY THE REV. E. H. ASKWITH

(Pp. 443, 1947, Price 21*sh*. 6*d*., Macmillan & Co., London)

This book was first published in 1908 and has passed through three editions, and the third edition has been reprinted four times already, all of which is evidence of the excellent qualities of the book. In fact, it has been in use in several Indian Universities as a text-book since its publication, with satisfactory results,

One feels disappointed, however, that the various opportunities afforded by reprinting have not been utilized for effecting some improvements, such as a better grading of examples, avoidance or fusion of nearly equivalent questions (such as for example questions 1 and 31, 12 and 19 in chapter 9). A more important point which calls for attention is the treatment of the equation of the line at infinity in areals ($x + y + z = 0$) which has to be reconciled with the identical relation $x + y + z = 1$. The author's explanation on page 306 that in this case x, y, z are infinite but their sum is finite (being equal to 1), and that "if $x + y + z$ be finite it is the same as if we write $x + y + z = 0$, for finite quantities are negligible in comparison with infinite ones" sounds hardly convincing. No treatment of this subject will be free from contradictions which does not bring out the fact that points on a projective plane are defined in a way different from those of the cartesian plane (as the ratios of three numbers x, y, z not all zero), and that the projective plane contains all the points of the cartesian plane and in addition an extra line (line at infinity). In such a treatment there is no place for $x + y + z = 1$ for *all points* of the projective plane, nor any special importance attached to such a relation.

A. N. RAO.

Demlo Numbers

BY D. R. KAPREKAR, B.SC., S.T.C.

(Pp. iii+114, 1948, Price Rs. 6, Aryabhushan Press, Poona)

This is a book of Collected Papers, twenty in all, written mostly by D. R. Kaprekar himself, while a few are in collaboration with other authors, on a subject bearing a curious title which deserves some explanation. It is mentioned in the author's Preface that the word 'Demlo' is derived from the town Dombivli, a Railway Station about 30 miles from Bombay. The frequent occurrence of numbers like 165; 2553; 47773 on waggons, motors, tickets, etc., impressed on the young author's mind during his constant travels between Bombay and Dombivli the peculiar character of these numbers, *viz.*, the digits arranging themselves in symmetric compliments with respect to a single central digit which may be repeated any number of times. Soon these numbers took such a firm hold on his mind that he was led to study such numbers and the result is a series of arithmetical researches which were published mainly in the *Bombay University Journal* more or less in the decade 1938–1947. The author has classified these numbers into 12 types and illustrated the diverse arithmetical situations wherein they occur. The colourful names of these such as super-wonder Demlo Numbers and the arithmetical processes connected with them may raise a smile in modern minds, but they are an indication of the intense interest excited by them in the author's mind. The processes to which the Demlo numbers are subjected are squaring, cubing, etc., multiplication and diagonal summation so as to yield various kinds of curious-looking integers. Witness, for example, how 999^5 is derived from $9^5 = 59049$ by spreading out its digits and inserting in the gaps 99 and 00 alternately thus: 995009990004999.

One important application of diagonal summation is to get the digits of a recurring decimal, say 1/109, by adding together diagonally the numbers of a suitable recurring series, which in this case happens to be the Fibonacci Series. It is noteworthy that the last digits of the recurring decimal for 1/109, read backwards are just the terms of the Fibonacci Series 1, 1, 2, 3, 5, 8, 13 which retain their individuality up to the 6th term and thereafter get merged one with the other (see Chapter XIV). There is, however, nothing extraordinary about diagonal summations. Our ordinary multiplications also involve diagonal summations. Another example of diagonal summation is the set of triangular numbers 1, 3, 6, 10, 15, ... which yield the extreme right digits of the recurring decimal for 1/729. What is important to see is how division is replaced in this process by mere addition, which can be readily done on a calculating machine.

It is impossible to do full justice to the curious and striking properties of Demlo Numbers in a short review and the interested reader should read up the book for further details. The typography is good and most of the mistakes are corrected in the Errata.

I have seen Mr. Kaprekar giving an exhibition of Demlo Magic before student audiences, and still remember the excitement and repeated cheers it calls forth—unlike anything connected with a mathematical lecture. I commend this collection of papers to all laymen and junior mathematical workers as well as school and college libraries and cultural associations.

A. N. RAO.

28 JAN 1950
MADRAS

FIFTEENTH CONFERENCE

Third Symposium on Applied Mathematics: Elasticity

The American Mathematical Society now holds annual Symposiums on selected subjects in Applied Mathematics. The subject of the first symposium, held in August, 1947, was Non-linear problems in Mechanics of Continua. In 1948 the subject of the second symposium was Electromagnetic Theory. Both these symposiums are being published in book form.

The third annual symposium was held at the University of Michigan, Ann Arbor, Michigan, from the 14th of June to the 16th of June, 1949. The subject of the symposium was Elasticity. The Applied Mechanics division of the American Society of Mechanical Engineers was a co-sponsor of the symposium. The Proceedings of this symposium are to be published in a book form by McGraw-Hill.

This symposium was very well attended. On the second day about 300 mathematicians and engineers from all parts of the United States were present. There were seventeen invited papers and addresses. The chief contributors included Sir Richard Southwell, Professor E. Reissner, Professor I. S. Sokolnikoff, Professor W. Prager, Professor D. L. Holl and Professor B. R. Seth. In the Applied Mechanics division Professor S. Woinowsky-Krieger, Mr. H. Poritsky, Professor D. C. Drucker and Professor L. H. Donnell were amongst the contributors.

Sir Richard Southwell showed how his relaxation method could be used for the elasto-plastic torsion problem. Professor E. Reissner, Professor F. B. Hildebrand and Professor K. O. Friedrichs dealt with finite deflection theory of plates and shells. Professor I. S. Sokolnikoff gave an account of methods used to solve problems in anistropic elasticity. He showed how the perturbation method could be used to get approximate results. Professor B. R. Seth pointed out that the method implied that the anistropic elastic constants could be obtained from the istropic ones by adding small terms, which was not always possible. Professor D. L. Holl discussed the bending of anistropic plates under dynamic loads.

Professor W. Prager, Professor D. C. Drucker, Professor N. Coburn, and Mr. P. G. Hodge dealt with plastic problems. Professor B. R. Seth of Hindu College, Delhi, and at present Visiting Professor of Applied Mathematics at Iowa State College, discussed some recent applications of the theory of finite elastic deformations. He criticized some papers of P. M. Riz, N. Zvolinsky, R. Kappus and R. S. Rivlin. His suggestion that in the non-linear elastic domain the elastic constants should be reduced was very well borne out by papers of E. A. Davis, A. Eyering and G. Halsey, an account of which was given by C. J. Thorne. It was suggested to Professor L. H. Donnell that if these reduced values of the constants were used the wide disagreement between theory and his experimental results on buckling of thin cylinders under axial compression would become much less. Professor B. R. Seth also pointed out to Professor G. E. Hay

that, in his problem of the elliptic plate under concentrated load, the use of certain relations involving stresses in orthogonal curvilinear coördinates would enable him to obtain the solution in a closed form.—(Information kindly sent by Prof. B. R. Seth.)

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